Problems with interpretation of ^{10}He ground state

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The continuum of ^{10}He nucleus is studied theoretically in a three-body ^{8}He+n+n model basing on the recent information concerning ^{9}He spectrum [Golovkov, *et al.*, nucl-ex/0608035, submitted to Phys. Rev. C]. The ground state (g.s.) of ^{10}He for new g.s. energy of ^{9}He is found at about 2 MeV. The peak in the cross section may be shifted to a lower energy (e.g. $\sim$ 1.2 MeV) when ^{10}He is populated in reactions with ^{11}Li due to peculiar reaction mechanism. Possibility of the near-threshold g.s. of ^{10}He with $[s_{1/2}]^2$ structure is practically excluded by our calculations and a limits on the strength of s-wave interaction in the ^{9}He (^{8}He+n channel) are thus imposed.

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Introduction. — The first (theoretical) attempts to study ^{10}He were undertaken in the end of 60-th [1]. Nevertheless, ^{10}He remains relatively poorly studied system. Since it became clear that ^{10}He is nuclear unstable [2] and ground state properties of ^{9}He were defined [3, 4] it became possible to predict theoretically the ground state of ^{10}He as a narrow three-body resonance ($E \sim 0.7-0.9$, $\Gamma \sim 0.1-0.3$ MeV [5]) with valence neutrons populating mainly $[p_{1/2}]^2$ configuration. These predictions were soon confirmed experimentally: $E = 1.2(3)$, $\Gamma < 1.2$ MeV [6], $E = 1.07(7)$, $\Gamma = 0.3(2)$ MeV [7], and $E = 1.7 \pm 0.3 \pm 0.3$ MeV [8].

A new possible understanding of ^{10}He was proposed after existence of a virtual state in ^{9}He was shown in [9] (suggested limit for scattering length $a < -10$ fm). With such an attractive s-wave interaction a narrow near-threshold 0^+ state ($E = 0.05$, $\Gamma = 0.21$ MeV) with structure $[s_{1/2}]^2$ was predicted in ^{10}He in addition to $[p_{1/2}]^2$ 0^+ state calculated to be at about 1.7 MeV [10]. Concerning evident discrepancy with experimental data the author of [10] suggested that the ground state of ^{10}He just had not been observed so far and the state at ~ 1.3 MeV is actually the first excited. However, no possible explanation was proposed in Ref. [10] for which reason the $[s_{1/2}]^2$ g.s. was missed in experiments.

In the recent experiment on the Dubna radioactive beam facility ACCULINNA the low-lying spectrum of ^{9}He was revised, providing higher than in the previous studies position of $p_{1/2}$ state. A broad $p_{1/2}$ state was observed at about 2 MeV by Golovkov *et al.* in Ref. [11] instead of $p_{1/2}$-$p_{3/2}$ doublet of narrow states at 1.27 and 2.4 MeV as in Refs. [3, 4, 7]. This experiment also claim a unique spin-parity identification below 5 MeV. Presence of the $s_{1/2}$ contribution is evident in these data, but exact nature of this contribution is still unclear, whether it is virtual state with considerably large negative scattering length a or just a smooth nonresonant cross section. This new data should have a strong impact on the calculated properties of ^{10}He, which inspired us to "revisit" the issue.

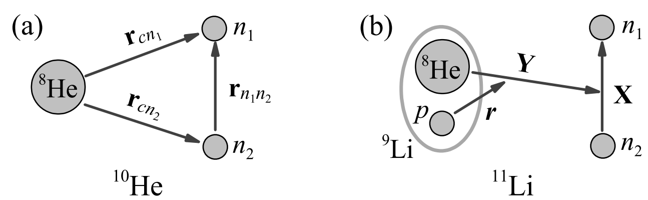

FIG. 1: Coordinate sets used in this paper. Panel (b) illustrates proton removal from ^{11}Li as method to populate ^{10}He.

In contrast with approach of [10], which provided only energies and widths for the states, we are interested in observable consequences of $[s_{1/2}]^2$ and $[p_{1/2}]^2$ states "coexistence" in the ^{10}He spectrum. We study the question in theoretical model, which is schematic but has definite relevance to real possible reaction mechanisms of population of the ^{10}He continuum.

Theoretical model. — In our approach we generally follow the prescription of [5] only with appropriate modification of potentials. For core-n subsystem from the set of potentials tested in this work we selected one (denoted there as "I2"); other choice do not change qualitatively the result and quantitatively is quite close. The potential is parameterized by Gaussian formfactors

$$V^l_{c,ls}(r) = V^l_{c,ls} \exp[-r^2/r_0^2]$$

with width $r_0 = 3.4$ fm. The depths of d-wave potential $V_c^2 = -33$ MeV and (ls) component in p-wave $V^1_{ls} = 10$ are the same as in original paper. The inverse (ls) forces were used in Ref. [5] in p-wave to account for occupied $p_{3/2}$ subshell in ^{8}He core. Additional repulsive core is introduced in s-wave with $r_0(core) = 2.35$ fm and $V_c^0(core) = 75$ MeV. Central potential parameters in s- and p-waves V_c^0 and V_c^1 are being varied. In the n-n subsystem the realistic GPT potential [12] is used.

To study qualitatively possible influence of the reaction mechanism we follow the approach of paper [14] to exotic ^{5}H system. We introduce a compact source func-

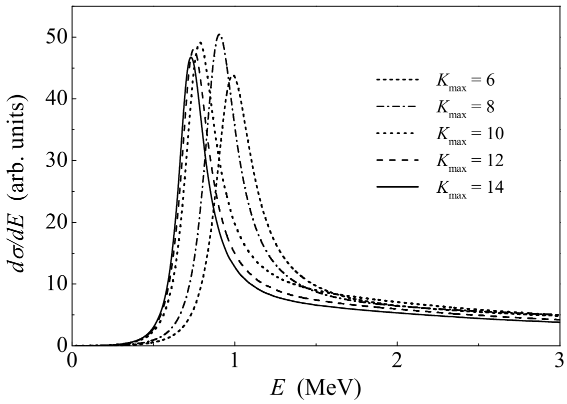

FIG. 2: Convergence of calculations as a function of $K_{\max}$ (value truncating the hyperspherical basis).

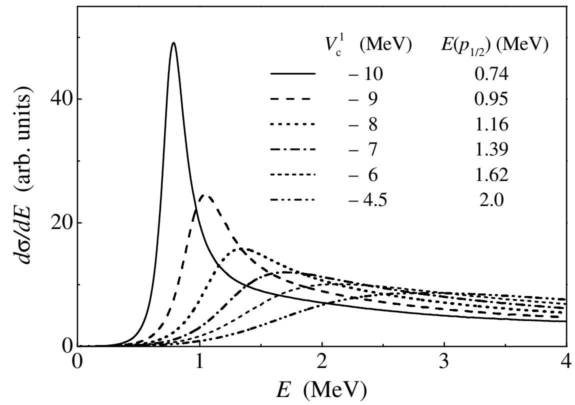

FIG. 3: Behaviour of the ^{10}He spectrum with decrease of p-wave potential depth. The corresponding $p_{1/2}$ state energies in ^{9}He relative to the threshold are shown in the legend.

tion $\Phi(\rho,\Omega_\rho)$ in the right hand side of the three-body Schrödinger equation and solve the inhomogeneous system of equations

$$\left(\hat{H}-E\right)\Psi_E^{(+)}(\rho,\Omega_\rho)=\Phi(\rho,\Omega_\rho)\,, \quad (1)$$

$$\hat{H}=\hat{T}+\hat{V}_{cn}(\mathbf{r}_{cn_1})+\hat{V}_{cn}(\mathbf{r}_{cn_2})+\hat{V}_{nn}(\mathbf{r}_{n_1n_2})\,,$$
$$\rho^2=\frac{8}{10}\left(r^2_{cn_1}+r^2_{cn_2}\right)+\frac{1}{10}r^2_{n_1n_2}=\frac{1}{2}X^2+\frac{8}{5}Y^2\,, \quad (2)$$

for pure outgoing wave boundary conditions (coordinates are shown in Fig. 1). The hyperradial components $\chi^{(+)}_{K\gamma}(\rho)$ of the WF (value $K_{\max}$ truncates the hyperspherical expansion)

$$\Psi_E^{(+)}(\rho,\Omega_\rho)=\rho^{-5/2}\sum_{K\gamma}^{K_{\max}}\chi^{(+)}_{K\gamma}(\varkappa\rho)\,\mathcal{J}^{JM}_{K\gamma}(\Omega_\rho)\,,$$

are matched to Riccati-Bessel functions of half-integer index $\mathcal{H}^{(+)}_{K+3/2}$, with asymptotic $\exp(i\varkappa r)$, $\varkappa=\sqrt{2ME}$, describing the partial outgoing waves for hyperspherical equations.

We used two different sources. One is "narrow" with a Gaussian formfactor

$$\Phi(\rho,\Omega_\rho)=\exp[-\rho^2/\rho_0^2]\,\sum_{K\gamma}\mathcal{J}^{JM}_{K\gamma}(\Omega_\rho)\,, \quad (3)$$

where $\rho_0=4.1$ fm provides the source rms radius $\langle\rho\rangle=5$ fm. This is a typical radius for the "reaction volume" with ordinary nuclei. This kind of source has a "generic" character; there is no selectivity and various components of the WF are populated. This is not very realistic, but guarantees that important peculiarities of the continuum WF are not missed. This approach is analogous to the approach of Ref. [13] [see Eq. (2) of this work] to studies of the ^{5}H continuum.

The other choice of source is reaction specific. When ^{10}He is produced from ^{11}Li in a process which can be approximated as a sudden removal of proton from ^{9}Li core, the source term $\Phi(\rho,\Omega_\rho)$ should contain the Fourier transform of the overlap integral between the ^{8}He WF $\Psi_{^8\mathrm{He}}$, spin-isospin function of the removed proton χ_p and the ^{11}Li wave function over the radius-vector $\mathbf{r}$ between the removed proton and the center-of-mass of ^{10}He (see Fig. 1b):

$$\Phi(\rho,\Omega_\rho)=\int d\mathbf{r}\,e^{i\mathbf{qr}}\langle\Psi_{^8\mathrm{He}}\chi_p|\Psi_{^{11}\mathrm{Li}}\rangle\,. \quad (4)$$

In general, this quantity should be a complicated function of the vector of the recoil momentum q, transferred to the ^{10}He system in the proton removal process. However, if the reaction energy is large and the internal energy of ^{10}He is small this dependence could be neglected. Formal details of the approach can be found in [14]. It can be shown that partial hyperspherical components of the source function are well approximated by the corresponding hyperradial components of ^{11}Li WF. Thus, this type of calculations is further referred as "^{11}Li source". The ^{11}Li WF was taken from analytical parameterization [15] taking into account broad range of experimental information on this nucleus. The s^2 and p^2 configurations are populated by the ^{11}Li source with almost equal probabilities. The rms radius of such source function $\langle\rho\rangle=8.5$ fm is enormous compared to typical nuclear sizes.

The cross section for population of the ^{10}He continuum is proportional to the outgoing flux of the three particles on a hypersphere of some large radius $\rho=\rho_{\max}$:

$$\frac{d\sigma}{dE}\sim\frac{1}{M}\,\mathrm{Im}\int d\Omega_\rho\;\Psi_E^{(+)\dagger}\rho^{5/2}\frac{d}{d\rho}\,\rho^{5/2}\,\Psi_E^{(+)}\bigg|_{\rho=\rho_{\max}}\,. \quad (5)$$

The cross section convergence with the increase of the hyperspherical basis size is demonstrated in Fig. 2. The exponential convergence trend predicts that the result should converge some 40–60 keV below the $K_{\max}=14$ curve. For calculations of this work we chosen $K_{\max}=10$ which is close to converged result and has sufficient precision for our, mainly qualitative, studies.

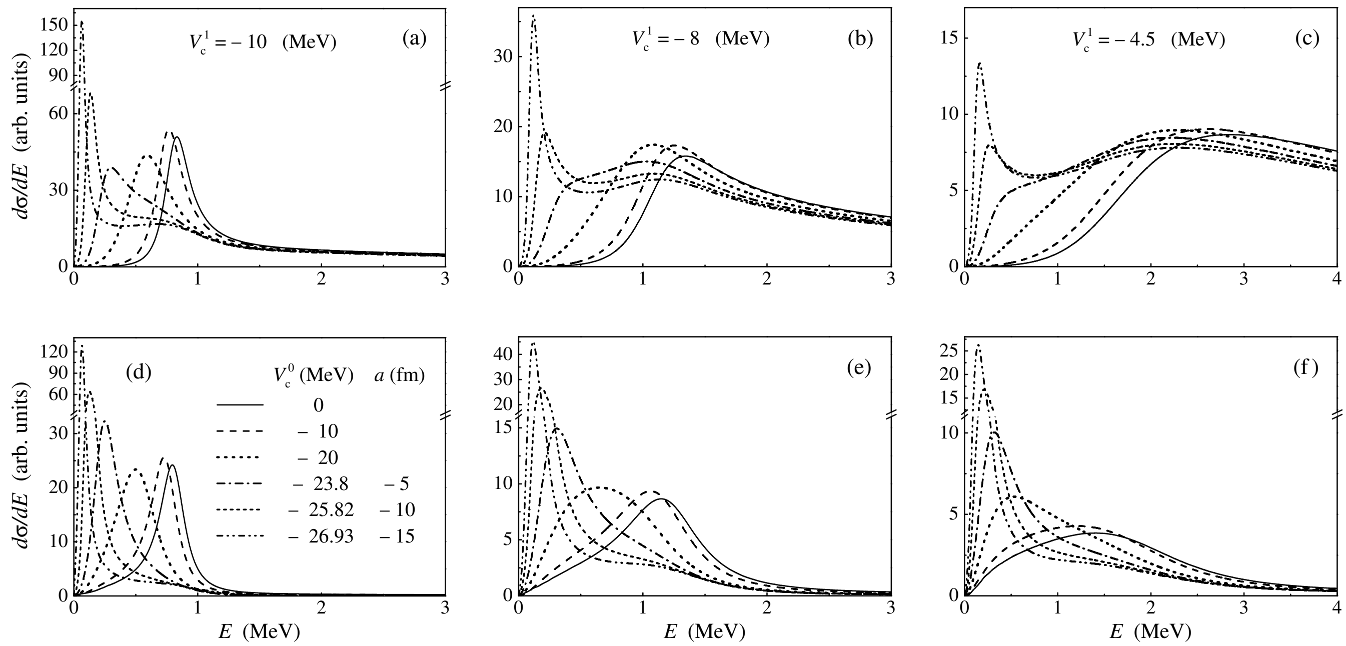

FIG. 4: Behaviour of the ^{10}He spectrum with decrease of s-wave interaction [legend, the same for all panels is shown in panel (d)] . First row shows the narrow source case, second row is for broad (^{11}Li) source. In the first column calculation for p-wave potential $V_c^1 = -10$ MeV ($p_{1/2}$ state at 074 MeV, as in calculations of [5]); in the second column for $V_c^1 = -8$ MeV ($p_{1/2}$ state at 1.16 MeV, close to 1.27 MeV, as in experiment [7]); in the third column for $V_c^1 = -4.5$ MeV ($p_{1/2}$ state at 2 MeV, as in experiment [11]).

Calculations. — With the described model we first of all reproduce the results of Ref. [5] (Fig. 3, solid curve). The evolution of cross section with decrease of p-wave interaction from the value adopted in [5] to a value providing the ^{9}He g.s. at about 2 MeV is shown in Fig. 3 for the narrow source function.

The evolution of cases with different p-wave interactions with increase of the s-wave interaction is shown in Fig. 4 for the narrow and broad source functions. The following facts should be noted:

(i) The narrow ground state in ^{10}He is not significantly sensitive to the reaction mechanism [Fig. 4 (a) and (d)]. When the state is above 1 MeV the dependence on the source function is considerable [(b) and (e)]. In the case of even higher ^{10}He g.s. the calculations with narrow and broad sources have very little in common [(c) and (f)]. A question can be asked in the latter case what should be considered as a "real" position of ^{10}He g.s. if such a diverse experimental responses could be expected. Fig. 5 shows the results of the $3 \rightarrow 3$ scattering calculations (see Ref. [13] for details of approach on example of ^{5}H nucleus). Three body Hamiltonian here is the same as in Fig. 4c,f. Three different values defined in these calculations (diagonal $3 \rightarrow 3$ scattering phase shifts, first diagonalized phase shift, and internal normalizations for scattering WF) indicate that peculiarity in the $3 \rightarrow 3$ scattering S-matrix is located at about 2 MeV. This value can be regarded as real position of ^{10}He g.s. while difference between Figs. 4c and 4f characterize the scale of uncertainty imposed by reaction mechanism in this case.

(ii) For relatively strong s-wave interaction in ^{8}He-n subsystem (namely such that scattering length $a < -5$ fm), we unavoidably (means independently on the structure and reaction mechanism details) get a sharp peak in the cross section with energy less than 0.3 MeV and dominating $[s_{1/2}]^2$ configuration. This low-energy peaks could hardly be consistent with the experimental data [6] (e.g. Fig. 6, dashed curve). For that reason we would impose theoretical limit $a \geq -5$ fm, which reliability is limited only by quality of the experimental data [6].

(iii) The experimental cross section peaked at about 1.2 MeV could be consistent with some range of p-wave interactions for ^{11}Li source [Fig. 4, (e)–(f)]. This, however, is possible only for quite weak attractive part of s-wave potential: $V_c^0 > -20$ MeV. For such value of parameters the s-wave interaction is in general still effectively repulsive (due to a large repulsive core). This limitation is even more stringent then that in item (ii). These theoretical limits, are in a strong contradiction with the *upper* limit for scattering length ($a < -10$ fm) imposed in experiment [9]. This contradiction should be considered seriously as conclusions of Ref. [9] strongly rely on quite complicated theoretical interpretation as well. There is no contradiction between our result and data [11] where a *lower* limit $a > -20$ fm for scattering length is given.

Discussion. — It was proposed in Ref. [10] that the observed so far state of ^{10}He is not the ground but the first excited state with $[p_{1/2}]^2$ structure while the ground

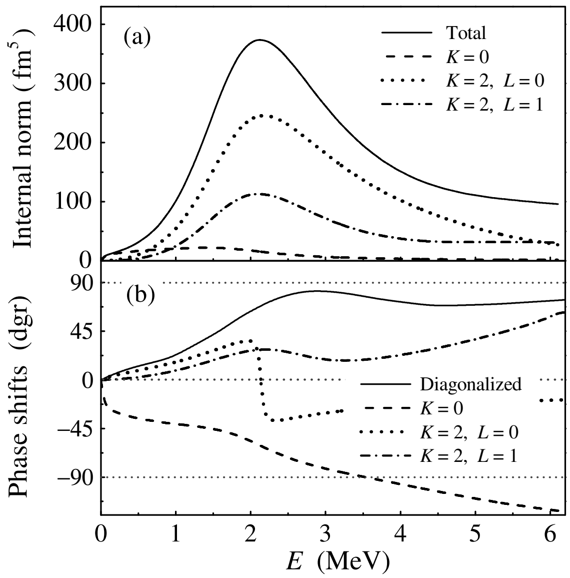

FIG. 5: The $3 \rightarrow 3$ scattering calculations. (a) Internal normalizations [Ref. [13], Eq. (1) with $\rho_0 = 6$ fm] for different components of the WF. (b) Diagonalized phase shift ("eigenphase") is shown by solid curve, while diagonal phase shifts for the lowest hyperspherical components are given by dashed, dotted, and dash-dotted curves.

$[s_{1/2}]^2$ remains unobserved. What we find here is that population of $[s_{1/2}]^2$ configuration in the case of strong s-wave attraction is always very pronounced and it is more likely that $[p_{1/2}]^2$ component is lost as a "nonresonant background" than vice versa. It can be found that position of $[p_{1/2}]^2$ component of the 0^+ state is quite stable when the s-wave attraction is increased. However, for extreme cases of s-wave attraction this contribution becomes much broader and in general "lost" on a thick right "tail" of the $[s_{1/2}]^2$ g.s. By certain mathematical procedure (e.g. as it was done in [10]) it is possible to extract properties of $[p_{1/2}]^2$ component as a distinct narrow state also in this situation. However, the schematic calculations taking even approximate care about reaction mechanism shows that from real data it would be practically impossible.

Several theoretical spectra of ^{10}He are provided in Fig. 6 on top of the experimental data Ref. [6]. Theoretical curves are convoluted with energy resolution of experiment which is parameterized as $\Delta E = 0.7\sqrt{E}$ (ΔE is FWHM). The calculation with ^{9}He subsystem having $p_{1/2}$ g.s. at about 2 MeV is most consistent with the data. However, some other cases (e.g. with ^{9}He g.s. at 1.2 MeV from Fig. 4b,e) can not be excluded. Thus, the information on ^{10}He spectrum itself may not be sufficient to define its properties. It should be used in conjunction with reaction model and reliable information on ^{9}He properties.

The situation with unclear reaction mechanism can be resolved by experiment with different reaction mechanism. For ^{10}He such experiment exist: the ground and

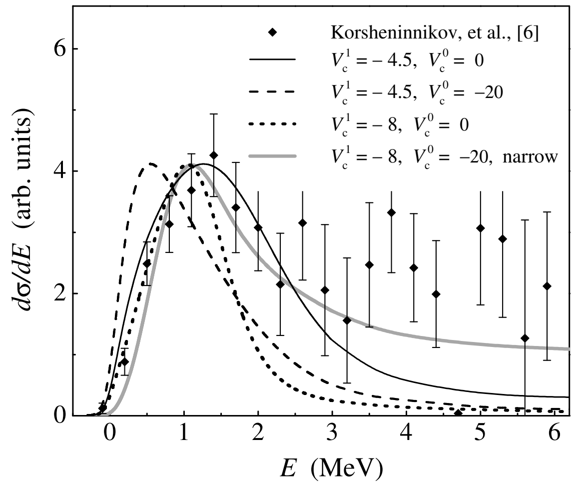

FIG. 6: Calculation results convoluted with experimental resolution of Ref. [6] and experimental data. Solid, dashed, and dotted curves correspond to calculations with ^{11}Li source [see Fig. 4 (f) solid, (f) dotted, and (e) solid curves]. Gray line shows calculation with narrow source [Fig. 4 (b) dotted curve].

two excited states were observed in [7]. Our calculations, however, are hardly possible to make consistent with its results. The small width of the g.s. obtained in this work (300 keV at 1.07 MeV of excitation) is not reproduced in any of our calculations.

The partial decompositions of the cross section given in Fig. 7 to show how the contributions of $[s_{1/2}]^2$ component (mainly $K = 0$) and $[p_{1/2}]^2$ component (mainly $K = 2$) change when we switch from the narrow to the broad source. In the first case both contributions has broad peak at about 2.5–3 MeV while in the case of ^{11}Li source the contributions become split (although not enough to make them distinguishable in the invariant mass spectrum of ^{10}He). The recent experience of the ^{5}H system studies [16, 17] shows that such picture could be decomposed in analysis of correlation data obtained in certain experimental technique (transfer reactions in "zero geometry").

The question can be raised, how reliable is the statement that for $p_{1/2}$ state in ^{9}He at about 2 MeV we can not get a *state* in ^{10}He at 1.2 MeV straightforwardly. In Table I we list paring energy for valence neutrons calculated for ^{10}He in different theoretical approaches. With

TABLE I: Paring energy (in MeV) for ^{10}He defined as $E_p = S_{2n} - 2S_n$ calculated in different theoretical approaches.

Work	[2]	[5][a]	[18]	[19]	[10]	[20][b]	This
$-S_{2n}$	1.18	0.8	1.09	2.78	1.68	1.94	2.0
E_p	1.26	0.75	0.59	1.98	0.86	1.25	2.0

[a] We use $p_{1/2}$ elastic cross section peak energy to define S_n.
[b] See Table 1, column 6.

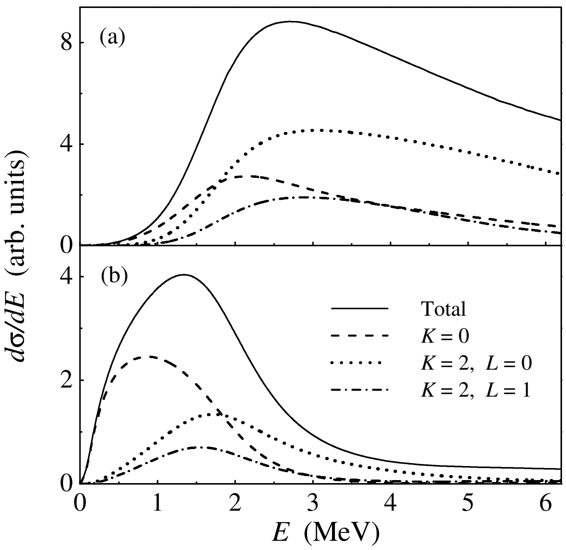

FIG. 7: Calculations with $V_c^0 = 0$, and $V_c^1 = -4.5$ MeV for narrow (a) and for ^{11}Li source (b). Dashed dotted and dash-dotted curves provide contributions of the main WF components.

$p_{1/2}$ state at 2 MeV this energy should be about 2.8 MeV, while in various theoretical approaches it does not exceed 2 MeV. It is clear that relatively small (say, compared to nearby ^{6}He and ^{8}He isotopes) paring energy in ^{10}He is common for different theoretical approaches and thus it could be a real situation.

Conclusion. — We would like to emphasize the following result of our studies:
(i) Within theoretical model for $p_{1/2}$ state in the ^{9}He located at about 2 MeV it is problematic to obtain the ^{10}He g.s. at about 1.2 MeV straightforwardly. The required for that paring energy is 2.8 MeV, while for $[p_{1/2}]^2$ configuration it is typically obtained $\sim 1-2$ MeV.
(ii) The attraction in s-wave does not allow to shift state with $[p_{1/2}]^2$ configuration to significantly lower energy. Instead, for some extreme values of attraction ($a \leq -5$ fm) lead to formation of low-energy $[s_{1/2}]^2$ state which forms a sharp peak in the cross section at energies less than 0.3 MeV. Appearance of such state is in accord with predictions of Ref. [10].
(iii) In contrast with predictions of Ref. [10], the state with $[p_{1/2}]^2$ structure in the presence of $[s_{1/2}]^2$ ground state either disappear or become very broad (small on the s-wave "background"). For that reason the idea of Ref. [10] that the ground $[s_{1/2}]^2$ state of the ^{10}He remains unobserved, while the observed so far state is the first excited one with $[p_{1/2}]^2$ structure, does not get support in our studies.

Concerning comparison with experimental data:
(i) Calculations with large negative scattering length (e.g. $a < -5$ fm) in core-n subsystem are not consistent with experimental data. In any calculated case we obtain for such situation a single narrow peak below 0.5 MeV which could have hardly been overlooked in experiment [6].
(ii) Observation of quite a broad peak at about 1.2 MeV in Ref. [6] could be explained by a specific mechanism of the chosen reaction induced by ^{11}Li (namely the huge size of neutron halo in ^{11}Li). For ^{10}He ground $[p_{1/2}]^2$ state located at $E \geq 2$ MeV this leads to a strong enhancement of the low-energy transition strength even without any significant attraction in s-wave. As a result, the peak in the cross section may be shifted to a lower energy (e.g. ~ 1.2 MeV).
(iii) The existing experimental data do not allow unambiguously establish the "real" g.s. position for ^{10}He. Alternative experiments (relative to those utilizing ^{11}Li beams) are desirable. Further clarification of controversy with the ^{9}He spectrum is indispensable for theoretical understanding of the ^{10}He properties.

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[1] A.I. Baz, V.F. Demin, and M.V. Zhukov, Sov. J. Nucl. Phys. **9**, 693 (1969) [Yad. Fiz. **9**, 1184 (1969)].
[2] J. Stevenson, *et al.*, Phys. Rev. C **37**, 2220 (1988).
[3] K. Seth, *et al.*, Phys. Rev. Lett. **58**, 1930 (1987).
[4] H. G. Bohlen, *et al.*, Z. Phys. **A330**, 227 (1988).
[5] A.A. Korsheninnikov, B.V. Danilin, and M.V. Zhukov, Nucl. Phys. **A559**, 208 (1993).
[6] A.A. Korsheninnikov, *et al.*, Phys. Lett. **B326**, 31 (1994).
[7] H.G. Bohlen, *et al.*, Prog. Part. Nucl. Phys. **42**, 17 (1999).
[8] T. Kobayashi, *et al.*, Nucl. Phys. **A616**, 223c (1997).
[9] L. Chen, *et al.*, Phys. Lett. **B505**, 21 (2001).
[10] S. Aoyama, Phys. Rev. Lett. **89**, 052501 (2004).
[11] M.S. Golovkov, *et al.*, nucl-ex/0608035, submitted to Phys. Rev. C.
[12] D. Gogny, P. Pires and R. de Tourreil, Phys. Lett. **B32**, 591 (1970).
[13] N.B. Shulgina, *et al.*, Phys. Rev. C **62**, 014312 (2000)
[14] L.V. Grigorenko, N.K. Timofeyuk, and M.V. Zhukov, Eur. Phys. J. **A 19**, 187 (2004).
[15] N.B. Shulgina and M.V. Zhukov, privat communication.
[16] M.S. Golovkov, *et al.*, Phys. Rev. Lett. **93** 262501 (2004).
[17] M.S. Golovkov, *et al.*, Phys. Rev. C **72** 064612 (2005).
[18] Y. Shen and Z. Ren, Phys. Rev. C **54** 1158 (1996).
[19] P. Navratil and B.R. Barrett, Phys. Rev. C **57** 3119 (1998).
[20] A. Volya and V. Zelevinsky, Phys. Rev. Lett. **94**, 052501 (2005).